## cmareren 25 Real Options

Honda Motor Company spent $\$ 400$ million dollars on something it might not need-production flexibility. If demand for its vehicles changed in predictable ways, then Honda would have wasted the $\$ 400$ million dollars. But as we have seen in the global economic recession, demand for automobiles is highly volatile, with consumer preferences swinging wildly every time oil prices change. To prepare for such volatility, Honda has been building flexibility into its factories and now boasts the most flexibility of any auto maker in the United States.

Honda's plant in Liberty, Ohio, can stop Civic production, set up for the CR-V crossover, and start producing CR-Vs in less than 10 minutes, incurring virtually no additional cost in the process. Many of its other plants have similar capabilities. For example, Honda has been able to quickly reduce output of its Ridgeline pickup truck and boost production of more fuel-efficient vehicles. In contrast, Ford will take over a year to convert a factory now producing gas-guzzling sport utility vehicles, with the switchover costing over $\$ 75$ million. GM has similar problems and will spend $\$ 370$ million to change models at one of its factories.

Honda's flexibility is due to several factors, beginning with designs for vehicles and production processes that share components and assembly techniques. For example, the assembly process for doors is very similar, no matter what vehicle is being produced. Honda's robots also give it flexibility. For example, the same robots are used to weld different vehicles.

It costs more initially to build in flexibility at a factory, but the payoff can be well worth the cost. As you read this chapter and learn more about options, think about how option pricing techniques can lead to better capital budgeting decisions.

[^0]The textbook's Web site contains an Excel file that will guide you through the chapter's calculations. The file for this chapter is Ch25 Tool Kit.xls, and we encourage you to open the file and follow along as you read the chapter.

Traditional discounted cash flow (DCF) analysis-in which an asset's cash flows are estimated and then discounted to obtain the asset's NPV-has been the cornerstone for valuing all types of assets since the 1950s. Accordingly, most of our discussion of capital budgeting has focused on DCF valuation techniques. However, in recent years academics and practitioners have demonstrated that DCF valuation techniques do not always tell the complete story about a project's value and that rote use of DCF can, at times, lead to incorrect capital budgeting decisions. ${ }^{1}$

DCF techniques were originally developed to value securities such as stocks and bonds. Securities are passive investments: Once they have been purchased, most investors have no influence over the cash flows the assets produce. However, real assets are not passive investments because managerial actions after an investment has been made can influence its results. Furthermore, investing in a new project often brings with it the potential for increasing the firm's future investment opportunities. Such opportunities are, in effect, options-the right (but not the obligation) to take some action in the future. As we demonstrate in the next section, options are valuable, so projects that expand the firm's set of opportunities have positive option values. Similarly, any project that reduces the set of future opportunities destroys option value. Since a project's impact on the firm's opportunities, or its option value, may not be captured by conventional NPV analysis, this option value should be considered separately, as we do in this chapter.

### 25.1 Valuing Real Options

Recall from Chapter 11 that real options are opportunities for management to change the timing, scale, or other aspects of an investment in response to changes in market conditions. These opportunities are options in the sense that management can, if it is in the company's best interest, undertake some action; management is not required to undertake the action. These opportunities are real (as opposed to financial) because they involve decisions regarding real assets-such as plants, equipment, and land-rather than financial assets like stocks or bonds. Four examples of real options are investment timing options, growth options, abandonment options, and flexibility options. This chapter provides an example of how to value an investment timing option and a growth option. Web Extension 25A on the textbook's Web site shows how to value an abandonment option.

Valuing a real option requires judgment, both to formulate the model and to estimate the inputs. Does this mean the answer won't be useful? Definitely not. For example, the models used by NASA only approximate the centers of gravity for the moon, the earth, and other heavenly bodies, yet even with these "errors" in their models, NASA has been able to put astronauts on the moon. As one professor said, "All models are wrong, but some are still quite useful." This is especially true for real options. We might not be able to find the exact value of a real option, but the value we find can be helpful in deciding whether or not to accept the project. Equally important, the process of looking for and then valuing real options often identifies critical issues that might otherwise go unnoticed.

Five possible procedures can be used to deal with real options. Starting with the simplest, they are as follows.

[^1]All calculations for the analysis of the investment timing option are shown in Ch25 Tool Kit.xls on the textbook's Web site.

1. Use discounted cash flow (DCF) valuation and ignore any real options by assuming their values are zero.
2. Use DCF valuation and include a qualitative recognition of any real option's value.
3. Use decision-tree analysis.
4. Use a standard model for a financial option.
5. Develop a unique, project-specific model using financial engineering techniques.

The following sections illustrate these procedures.
List the five possible procedures for dealing with real options.

### 25.2 The Investment Timing Option: An Illustration

There is frequently an alternative to investing immediately-the decision to invest or not can be postponed until more information becomes available. By waiting, a betterinformed decision can be made, and this investment timing option adds value to the project and reduces its risk.

Murphy Systems is considering a project for a new type of handheld device that provides wireless Internet connections. The cost of the project is $\$ 50$ million, but the future cash flows depend on the demand for wireless Internet connections, which is uncertain. Murphy believes there is a $25 \%$ chance that demand for the new device will be high, in which case the project will generate cash flows of $\$ 33$ million each year for 3 years. There is a $50 \%$ chance of average demand, with cash flows of $\$ 25$ million per year, and a $25 \%$ chance that demand will be low and annual cash flows will be only $\$ 5$ million. A preliminary analysis indicates that the project is somewhat riskier than average, so it has been assigned a cost of capital of $14 \%$ versus $12 \%$ for an average project at Murphy Systems. Here is a summary of the project's data:

| Demand | Probability | Annual Cash Flow |
| :--- | :---: | :---: |
| High | 0.25 | $\$ 33$ million |
| Average | 0.50 | 25 million |
| Low | 0.25 | $\underline{5 \text { million }}$ |
| Expected annual cash flow |  | million |
| Project's cost of capital <br> Life of project <br> Required investment, <br> or cost of project | $\$ 50$ million |  |

Murphy could accept the project and implement it immediately; however, since the company has a patent on the device's core modules, it could also choose to delay the decision until next year, when more information about demand will be available. The cost will still be $\$ 50$ million if Murphy waits, and the project will still be expected to generate the indicated cash flows, but each flow will be pushed back 1 year. However, if Murphy waits then it will know which of the demand conditionsand hence which set of cash flows-will obtain. If Murphy waits then it will, of course, make the investment only if demand is sufficient to yield a positive NPV.

Observe that this real timing option resembles a call option on a stock. A call gives its owner the right to purchase a stock at a fixed strike price, but only if the stock's price is higher than the strike price will the owner exercise the option and buy the stock. Similarly, if Murphy defers implementation, then it will have the right to
"purchase" the project by making the $\$ 50$ million investment if the NPV as calculated next year, when new information is available, is positive.

## Approach 1. DCF Analysis Ignoring the Timing Option

Based on probabilities for the different levels of demand, the expected annual cash flows are $\$ 22$ million per year:

$$
\begin{aligned}
\text { Expected cash flow per year } & =0.25(\$ 33)+0.50(\$ 25)+0.25(\$ 5) \\
& =\$ 22 \text { million }
\end{aligned}
$$

Ignoring the investment timing option, the traditional NPV is $\$ 1.08$ million, found as follows:

$$
\mathrm{NPV}=-\$ 50+\frac{\$ 22}{(1+0.14)^{1}}+\frac{\$ 22}{(1+0.14)^{2}}+\frac{\$ 22}{(1+0.14)^{3}}=\$ 1.08
$$

The present value of the cash inflows is $\$ 51.08$ million while the cost is $\$ 50$ million, leaving an NPV of $\$ 1.08$ million.

Based just on this DCF analysis, Murphy should accept the project. Note, however, that if the expected cash flows had been slightly lower-say, $\$ 21.5$ million per year-then the NPV would have been negative and the project would have been rejected. Also, note that the project is risky: there is a $25 \%$ probability that demand will be weak, in which case the NPV will turn out to be a negative $\$ 38.4$ million.

## Approach 2. DCF Analysis with a Qualitative Consideration of the Timing Option

The discounted cash flow analysis suggests that the project should be accepted, but just barely, and it ignores the existence of a possibly valuable real option. If Murphy implements the project now, it gains an expected (but risky) NPV of $\$ 1.08$ million. However, accepting now means that it is also giving up the option to wait and learn more about market demand before making the commitment. Thus, the decision is this: Is the option Murphy would be giving up worth more or less than $\$ 1.08$ million? If the option is worth more than $\$ 1.08$ million then Murphy should not give up the option, which means deferring the decision-and vice versa if the option is worth less than $\$ 1.08$ million.

Based on the discussion of financial options in Chapter 8, what qualitative assessment can we make regarding the option's value? Put another way: Without doing any additional calculations, does it appear that Murphy should go forward now or wait? In thinking about this decision, first note that the value of an option is higher if the current value of the underlying asset is high relative to its strike price, other things held constant. For example, a call option with a strike price of $\$ 50$ on a stock with a current price of $\$ 50$ is worth more than if the current price were $\$ 20$. The strike price of the project is $\$ 50$ million, and our first guess at the value of its cash flows is $\$ 51.08$ million. We will calculate the exact value of Murphy's underlying asset later, but the DCF analysis does suggest that the underlying asset's value will be close to the strike price, so the option should be valuable. We also know that an option's value is higher the longer its time to expiration. Here the option has a 1-year life, which is fairly long for an option, and this also suggests that the option is probably valuable. Finally, we know that the value of an option increases with the risk of the underlying asset. The data used in the DCF analysis indicate that the project is quite risky, which again suggests that the option is valuable.

Thus, our qualitative assessment indicates that the option to delay might well be more valuable than the expected NPV of $\$ 1.08$ if we undertake the project immediately. This conclusion is quite subjective, but the qualitative assessment suggests that Murphy's management should go on to make a quantitative assessment of the situation.

## Approach 3. Scenario Analysis and Decision Trees

Part 1 of Figure $25-1$ presents a scenario analysis and decision tree similar to the examples in Chapter 11. Each possible outcome is shown as a "branch" on the tree. Each branch shows the cash flows and probability of a scenario laid out as a time line. Thus, the top line, which gives the payoffs of the high-demand scenario, has positive cash flows of $\$ 33$ million for the next 3 years, and its NPV is $\$ 26.61$ million. The average-demand branch in the middle has an NPV of $\$ 8.04$ million, while the NPV of the low-demand branch is a negative $\$ 38.39$ million. Since Murphy will suffer a

## FIGURE 25-1 DCF and Decision-Tree Analysis for the Investment Timing Option (Millions of Dollars)



[^2]$\$ 38.39$ million loss if demand is weak and since there is a $25 \%$ probability of weak demand, the project is clearly risky.

The expected NPV is the weighted average of the three possible outcomes, where the weight for each outcome is its probability. The sum in the last column in Part 1 shows that the expected NPV is $\$ 1.08$ million, the same as in the original DCF analysis. Part 1 also shows a standard deviation of $\$ 24.02$ million for the NPV and a coefficient of variation (defined as the ratio of standard deviation to the expected NPV) of 22.32, which is rather large. Clearly, the project is quite risky under the analysis thus far.

Part 2 is set up similarly to Part 1 except that it shows what happens if Murphy delays the decision and then implements the project only if demand turns out to be high or average. No cost is incurred now at Year 0-here the only action is to wait. Then, if demand is average or high, Murphy will spend $\$ 50$ million at Year 1 and receive either $\$ 33$ million or $\$ 25$ million per year for the following 3 years. If demand is low, as shown on the bottom branch, Murphy will spend nothing at Year 1 and will receive no cash flows in subsequent years. The NPV of the high-demand branch is $\$ 23.35$ million and that of the average-demand branch is $\$ 7.05$ million. Because all cash flows under the low-demand scenario are zero, the NPV in this case will also be zero. The expected NPV if Murphy delays the decision is $\$ 9.36$ million.

This analysis shows that the project's expected NPV will be much higher if Murphy delays than if it invests immediately. Also, since there is no possibility of losing money under the delay option, this decision also lowers the project's risk. This plainly indicates that the option to wait is valuable; hence Murphy should wait until Year 1 before deciding whether to proceed with the investment.

Before we conclude the discussion of decision trees, note that we used the same cost of capital, $14 \%$, to discount cash flows in the "proceed immediately" scenario analysis in Part 1 and under the "delay 1 year" scenario in Part 2. However, this is not appropriate for three reasons. First, since there is no possibility of losing money if Murphy delays, the investment under that plan is clearly less risky than if Murphy charges ahead today. Second, the $14 \%$ cost of capital might be appropriate for risky cash flows, yet the investment in the project at Year 1 in Part 2 is known with certainty. Perhaps, then, we should discount it at the risk-free rate. ${ }^{2}$ Third, the project's cash inflows (excluding the initial investment) are different in Part 2 than in Part 1 because the low-demand cash flows are eliminated. This suggests that if $14 \%$ is the appropriate cost of capital in the "proceed immediately" case then some lower rate would be appropriate in the "delay decision" case.

In Figure 25-2, Part 1, we repeat the "delay decision" analysis but with one exception. We continue to discount the operating cash flows in Year 2 through Year 4 at the $14 \%$ WACC, but now we discount the project's cost at Year 1 using the risk-free rate of $6 \%$. This increases the PV of the cost, which lowers the NPV from $\$ 9.36$ million to $\$ 6.88$ million. Yet we really don't know the precise WACC for this project-the $14 \%$ we used might be too high or too low for the operating cash flows in Year 2 through Year 4. ${ }^{3}$ Therefore, in Part 2 of Figure 25-2 we show a sensitivity analysis of the NPV

[^3]FIGURE 25-2 Decision Tree and Sensitivity Analysis for the Investment Timing Option (Millions of Dollars)


[^4]in which the discount rates used for both the operating cash flows and for the project's cost vary. This sensitivity analysis shows that, under all reasonable WACCs, the NPV of delaying is greater than $\$ 1.08$ million. This confirms that the option to wait is more valuable than the $\$ 1.08$ million NPV resulting from immediate implementation. Therefore, Murphy should wait rather than implement the project immediately.

## Approach 4. Valuing the Timing Option with the Black-Scholes Option Pricing Model

The decision-tree approach, coupled with a sensitivity analysis, may provide enough information for a good decision. However, it is often useful to obtain additional insights into the real option's value, which means using the fourth procedure, an option pricing model. To do this, the analyst must find a standard financial option that resembles the
project's real option. ${ }^{4}$ As noted earlier, Murphy's option to delay the project is similar to a call option on a stock. Hence, the Black-Scholes option pricing model can be used. This model requires five inputs: (1) the risk-free rate, (2) the time until the option expires, (3) the strike price, (4) the current price of the stock, and (5) the variance of the stock's rate of return. Therefore, we need to estimate values for those five inputs.

First, if we assume that the rate on a 52 -week Treasury security is $6 \%$, then this rate can be used as the risk-free rate. Second, Murphy must decide within a year whether or not to implement the project, so there is 1 year until the option expires. Third, it will cost $\$ 50$ million to implement the project, so $\$ 50$ million can be used for the strike price. Fourth, we need a proxy for the value of the underlying asset, which in Black-Scholes is the current price of the stock. Note that a stock's current price is the present value of its expected future cash flows. For Murphy's real option, the underlying asset is the project itself, and its current "price" is the present value of its expected future cash flows. Therefore, as a proxy for the stock price we can use the present value of the project's future cash flows. And fifth, the variance of the project's expected return can be used to represent the variance of the stock's return in the Black-Scholes model.

Figure 25-3 shows how one can estimate the present value of the project's cash inflows. We need to find the current value of the underlying asset-that is, the project. For a stock, the current price is the present value of all expected future cash flows, including those that are expected even if we do not exercise the call option. Note also that the strike price for a call option has no effect on the stock's current

## FIGURE 25-3 <br> Estimating the Input for Stock Price in the Option Analysis of the Investment Timing Option (Millions of Dollars)



[^5][^6]price. ${ }^{5}$ For our real option, the underlying asset is the delayed project, and its current "price" is the present value of all its future expected cash flows. Just as the price of a stock includes all of its future cash flows, so should the present value of the project include all of its possible future cash flows. Moreover, since the price of a stock is not affected by the strike price of a call option, we ignore the project's "strike price," or cost, when we find its present value. Figure $25-3$ shows the expected cash flows if the project is delayed. The PV of these cash flows as of now (Year 0 ) is $\$ 44.80$ million, and this is the input we should use for the current price in the Black-Scholes model.

The last required input is the variance of the project's return. Three different approaches could be used to estimate this input. First, we could use judgment-an educated guess. Here we would begin by recalling that a company is a portfolio of projects (or assets), with each project having its own risk. Since returns on the company's stock reflect the diversification gained by combining many projects, we might expect the variance of the stock's returns to be lower than the variance of one of its average projects. The variance of an average company's stock return is about $12 \%$, so we might expect the variance for a typical project to be somewhat higher, say, $15 \%$ to $25 \%$. Companies in the Internet infrastructure industry are riskier than average, so we might subjectively estimate the variance of Murphy's project to be in the range of $18 \%$ to $30 \%$.

The second approach, called the direct method, is to estimate the rate of return for each possible outcome and then calculate the variance of those returns. First, Part 1 in Figure 25-4 shows the PV for each possible outcome as of Year 1, the time when the option expires. Here we simply find the present value of all future operating cash flows discounted back to Year 1, using the WACC of $14 \%$. The Year-1 present value is $\$ 76.61$ million for high demand, $\$ 58.04$ million for average demand, and $\$ 11.61$ million for low demand. Then, in Part 2, we show the percentage return from the current time until the option expires for each scenario, based on the $\$ 44.80$ million starting "price" of the project at Year 0 as calculated in Figure 25-3. If demand is high, we will obtain a return of $71.0 \%$ : $(\$ 76.61-\$ 44.80) / \$ 44.80=0.710=71.0 \%$. Similar calculations show returns of $29.5 \%$ for average demand and $-74.1 \%$ for low demand. The expected percentage return is $14 \%$, the standard deviation is $53.6 \%$, and the variance is $28.7 \%$. ${ }^{6}$

The third approach for estimating the variance is also based on the scenario data, but the data are used in a different manner. First, we know that demand is not really limited to three scenarios; rather, a wide range of outcomes is possible. Similarly, the stock price at the time a call option expires could take one of many values. It is reasonable to assume that the value of the project at the time when we must decide on undertaking it behaves similarly to the price of a stock at the time a call option expires. Under this assumption, we can use the expected value and standard deviation of the project's value to calculate the variance of its rate of return, $\sigma^{2}$, with this formula: ${ }^{7}$

$$
\sigma^{2}=\frac{\ln \left(C V^{2}+1\right)}{\mathrm{t}}
$$

[^7]
## FIGURE 25-4

## Estimating the Input for Variance in the Option Analysis of the Investment Timing Option (Millions of Dollars)



## Notes:

${ }^{\text {a }}$ The WACC is $14 \%$. The Year- 2 through Year-4 cash flows are discounted back to Year 1.
${ }^{\mathrm{b}}$ The standard deviation is calculated as explained in Chapter 6.
${ }^{\text {c }}$ The coefficient of variation is the standard deviation divided by the expected value.
${ }^{\mathrm{d}}$ The Year-0 price is the expected PV from Figure 25-3.
${ }^{e}$ The Year-1 PVs are from Part 1.
${ }^{\text {f }}$ The returns for each scenario are calculated as $\left(\mathrm{PV}_{\left.\text {Year 1 }_{1}-\text { Price }_{\text {Year } 0}\right) / \text { Price }_{\mathrm{Y}_{\text {ear }} 0} .}\right.$
${ }^{9}$ The variance of return is the standard deviation squared.
"The expected "price" at the time the option expires is taken from Part 1.
'The standard deviation of expected "price" at the time the option expires is taken from Part 1.

Here CV is the coefficient of variation of the underlying asset's price at the time the option expires, and t is the time until the option expires. Although the three outcomes in the scenarios represent a small sample of the many possible outcomes, we can still use the scenario data to estimate the variance that the project's rate of return would have if there were an infinite number of possible outcomes. For Murphy's project, this indirect method produces the following estimate of the variance of the project's return:

$$
\sigma^{2}=\frac{\ln \left(0.47^{2}+1\right)}{1}=0.20=20 \%
$$

(25-1a)

Which of the three approaches is best? Obviously, they all involve judgment, so an analyst might want to consider all three. In our example, all three methods produce similar estimates, but for illustrative purposes we will simply use $20 \%$ as our initial estimate for the variance of the project's rate of return.

In Part 1 of Figure 25-5, we calculate the value of the option to defer investment in the project based on the Black-Scholes model, and the result is $\$ 7.04$ million.

## FIGURE 25-5

Estimating the Value of the Investment Timing Option Using a Standard Financial Option (Millions of Dollars)


[^8]Since this is significantly higher than the $\$ 1.08$ million NPV under immediate implementation and since the option would be forfeited if Murphy goes ahead right now, we conclude as before that the company should defer the final decision until more information is available.

Because judgmental estimates were made at many points in the analysis, it would be useful to see how sensitive the final outcome is to certain of the key inputs. Therefore, in Part 2 of Figure 25-5 we show the sensitivity of the option's value to different estimates of the variance. It is reassuring to see that, for all reasonable estimates of variance, the option to delay remains more valuable than immediate implementation.

## Approach 5. Financial Engineering

Sometimes an analyst might not be satisfied with the results of a decision-tree analysis and cannot find a standard financial option that corresponds to the real option. In such a situation the only alternative is to develop a unique model for the specific real option being analyzed, a process called financial engineering. When financial engineering is applied on Wall Street, where it was developed, the result is a newly designed financial product. ${ }^{8}$ When it is applied to real options, the result is the value of a project that contains embedded options.

Although financial engineering was originally developed on Wall Street, many financial engineering techniques have been applied to real options during the last 10 years. We expect this trend to continue, especially in light of the rapid improvements in computer processing speed and spreadsheet software capabilities. One financial engineering technique is called risk-neutral valuation. This technique uses simulation, and we discuss it in Web Extension 25B. Most other financial engineering techniques are too complicated for a course in financial management, so we leave a detailed discussion of them to a specialized course.

What is a decision tree?
In a qualitative analysis, what factors affect the value of a real option?

### 25.3 The Growth Option: An Illustration

As we saw with the investment timing option, there is frequently an alternative to merely accepting or rejecting a static project. Many investment opportunities, if successful, lead to other investment opportunities. The production capacity of a successful product line can later be expanded to satisfy increased demand, or distribution can be extended to new geographic markets. A company with a successful name brand can capitalize on its success by adding complementary or new products under the same brand. These growth options add value to a project and explain, for example, why companies are flocking to make inroads into the very difficult business environment in China.

Kidco Corporation designs and manufactures products aimed at the pre-teen market. Most of its products have a very short life, given the rapidly changing tastes of pre-teens. Kidco is now considering a project that will cost $\$ 30$ million. Management believes there is a $25 \%$ chance that the project will "take off" and generate operating cash flows of $\$ 34$ million in each of the next 2 years, after which pre-teen tastes will change and the project will be terminated. There is a $50 \%$ chance of average demand, in which case cash flows will be $\$ 20$ million annually for 2 years. Finally,

[^9]there is a $25 \%$ chance that the pre-teens won't like the product at all, and it will generate cash flows of only $\$ 2$ million per year. The estimated cost of capital for the project is $14 \%$.

Based on its experience with other projects, Kidco believes it will be able to launch a second-generation product if demand for the original product is average or above. This second-generation product will cost the same as the first-generation product, $\$ 30$ million, and the cost will be incurred at Year 2. However, given the success of the first-generation product, Kidco believes that the second-generation product would be just as successful as the first-generation product.

This growth option resembles a call option on a stock, since it gives Kidco the opportunity to "purchase" a successful follow-on project at a fixed cost if the value of the project is greater than the cost. Otherwise, Kidco will let the option expire by not implementing the second-generation product.

The following sections apply the first four valuation approaches: (1) DCF, (2) DCF and qualitative assessment, (3) decision-tree analysis, and (4) analysis with a standard financial option.

## Approach 1. DCF Analysis Ignoring the Growth Option

Based on probabilities for the different levels of demand, the expected annual operating cash flows for the project are $\$ 19$ million per year:

$$
0.25(\$ 34)+0.50(\$ 20)+0.25(\$ 2)=\$ 19.00
$$

Ignoring the investment timing option, the traditional NPV is $\$ 1.29$ million:

$$
\mathrm{NPV}=-\$ 30+\frac{\$ 19}{(1+0.14)^{1}}+\frac{\$ 19}{(1+0.14)^{2}}=\$ 1.29
$$

Based on this DCF analysis, Kidco should accept the project.

## Approach 2. DCF Analysis with a Qualitative Consideration of the Growth Option

Although the DCF analysis indicates that the project should be accepted, it ignores a potentially valuable real option. The option's time to maturity and the volatility of the underlying project provide qualitative insights into the option's value. Kidco's growth option has 2 years until maturity, which is a relatively long time, and the cash flows of the project are volatile. Taken together, this qualitative assessment indicates that the growth option should be quite valuable.

## Approach 3. Decision-Tree Analysis of the Growth Option

Part 1 of Figure 25-6 shows a scenario analysis for Kidco's project. The top line, which describes the payoffs for the high-demand scenario, has operating cash flows of $\$ 34$ million for the next 2 years. The NPV of this branch is $\$ 25.99$ million. The NPV of the average-demand branch in the middle is $\$ 2.93$ million, and it is $-\$ 26.71$ million for the low-demand scenario. The sum in the last column of Part 1 shows the expected NPV of $\$ 1.29$ million. The coefficient of variation is 14.54 , indicating that the project is very risky.

Part 2 of Figure 25-6 shows a decision-tree analysis in which Kidco undertakes the second-generation product only if demand is average or high. In these scenarios, shown on the top two branches of the decision tree, Kidco will incur a cost of $\$ 30$ million at Year 2 and receive operating cash flows of either \$34 million or \$20 million for the next 2 years, depending on the level of demand. If the demand is low,

FIGURE 25-6 $\quad$ Scenario Analysis and Decision-Tree Analysis for the Kidco Project (Millions of Dollars)


[^10]shown on the bottom branch, Kidco has no cost at Year 2 and receives no additional cash flows in subsequent years. All operating cash flows (which do not include the cost of implementing the second-generation project at Year 2) are discounted at the WACC of $14 \%$. Because the $\$ 30$ million implementation cost is known, it is discounted at the risk-free rate of $6 \%$. As shown in Part 2 of Figure 25-6, the expected NPV is $\$ 4.70$ million, indicating that the growth option is quite valuable.

Sensitivity Analysis of the Kidco Decision-Tree Analysis in Figure 25-6 (Millions of Dollars)

|  | A | B | C | 0 | E | $F$ | G | H | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 389 | Cost of Capital Used to Discount the $\$ 30$ Million Implementation Cost in Year 2 of the Second-Generation Project |  |  |  |  |  |  |  |  |
| \%61 |  |  | 3.0\% | 4.0\% | 5.0\% | 6.0\% | 7.0\% | 8.0\% | 9.0\% |
| 368 | ᄃ | 8.0\% | \$10.96 | \$11.36 | \$11.76 | \$12.14 | \$12.51 | \$12.88 | \$13.23 |
| 8t | $\bigcirc \bigcirc$ | 9.0\% | \$9.61 | \$10.01 | \$10.41 | \$10.79 | \$11.16 | \$11.52 | \$11.88 |
| 354 | ㅎㅣㅇㅀ | 10.0\% | \$8.30 | \$8.71 | \$9.10 | \$9.49 | \$9.86 | \$10.22 | \$10.57 |
| 326 | $\bigcirc$ | 11.0\% | \$7.04 | \$7.45 | \$7.84 | \$8.23 | \$8.60 | \$8.96 | \$9.31 |
| 36 |  | 12.0\% | \$5.83 | \$6.23 | \$6.63 | \$7.01 | \$7.38 | \$7.75 | \$8.10 |
| 367 | 흉 | 13.0\% | \$4.65 | \$5.06 | \$5.45 | \$5.84 | \$6.21 | \$6.57 | \$6.92 |
| 350 | O | 14.0\% | \$3.52 | \$3.92 | \$4.32 | \$4.70 | \$5.07 | \$5.44 | \$5.79 |
| 350 | "ㅎ. | 15.0\% | \$2.42 | \$2.83 | \$3.22 | \$3.61 | \$3.98 | \$4.34 | \$4.69 |
| 300 | ¢ | 16.0\% | \$1.36 | \$1.77 | \$2.16 | \$2.54 | \$2.92 | \$3.28 | \$3.63 |
| 301 | $0 \stackrel{0}{0}$ | 17.0\% | \$0.33 | \$0.74 | \$1.13 | \$1.52 | \$1.89 | \$2.25 | \$2.60 |
| 372 |  | 18.0\% | -\$0.66 | -\$0.25 | \$0.14 | \$0.52 | \$0.90 | \$1.26 | \$1.61 |

Note:
${ }^{\text {a }}$ The operating cash flows do not include the $\$ 30$ million implementation cost of the second-generation project in Year 2.

The option itself alters the risk of the project, which means that $14 \%$ is probably not the appropriate cost of capital. Table $25-1$ presents the results of a sensitivity analysis in which the cost of capital for the operating cash flows varies from $8 \%$ to $18 \%$. The sensitivity analysis also allows the rate used to discount the implementation cost at Year 2 to vary from $3 \%$ to $9 \%$. The resulting NPV is positive for all reasonable combinations of discount rates.

## Approach 4. Valuing the Growth Option with the Black-Scholes Option Pricing Model

The fourth approach is to use a standard model for a corresponding financial option. As we noted earlier, Kidco's growth option is similar to a call option on a stock, so we will use the Black-Scholes model to find the value of the growth option. The time until the growth option expires is 2 years. The rate on a 2 -year Treasury security is $6 \%$, and this provides a good estimate of the risk-free rate. Implementing the project will cost $\$ 30$ million, which is the strike price.

The input for stock price in the Black-Scholes model is the current value of the underlying asset. For the growth option, the underlying asset is the secondgeneration project, and its current value is the present value of its cash flows. The calculations in Figure 25-7 show that this value is $\$ 24.07$ million. Because the strike price of $\$ 30$ million is greater than the current "price" of $\$ 24.07$ million, the growth option is currently out-of-the-money.

Figure 25-8 shows the estimates for the variance of the project's rate of return using the two methods described earlier in the chapter for the analysis of the investment timing option. The direct method, shown in Part 2 of the figure, produces an estimate of $17.9 \%$ for the variance of return. The indirect method, in Part 3, estimates the variance as $15.3 \%$. Both estimates are somewhat higher than the $12 \%$ variance of a typical company's stock return, which is consistent with the idea that a project's variance is higher than a stock's because of diversification effects. Thus, an estimated variance of $15 \%$ to $20 \%$ seems reasonable. We use an initial estimate of $15.3 \%$ in our initial application of the Black-Scholes model, shown in Part 1 of Figure 25-9.

## FIGURE 25-7 <br> Estimating the Input for Stock Price in the Growth Option Analysis of the Investment Timing Option (Millions of Dollars)



## Notes:

${ }^{\text {a }}$ The WACC is $14 \%$. All cash flows in this scenario are discounted back to Year 0.
${ }^{\mathrm{b}}$ The standard deviation is calculated as in Chapter 6.
${ }^{\text {c }}$ The coefficient of variation is the standard deviation divided by the expected value.

Using the Black-Scholes model for a call option, Figure 25-9 shows a $\$ 4.34$ million value for the growth option. The total NPV is the sum of the first-generation project's NPV and the value of the growth option: Total NPV $=\$ 1.29+\$ 4.34=\$ 5.63$ million, which is much higher than the NPV of the first-generation project alone. As this analysis shows, the growth option adds considerable value to the original project. In addition, the sensitivity analysis in Part 2 of Figure 25-9 indicates that the growth option's value is large for all reasonable values of variance. Kidco should therefore accept the project.

For an illustrative valuation of an abandonment option, see Web Extension 25 A.
Explain how growth options are like call options.

### 25.4 Concluding Thoughts on Real Options

We don't deny that real options can be pretty complicated. Keep in mind, however, that 50 years ago very few companies used NPV because it seemed too complicated. Now NPV is a basic tool used by virtually all companies and taught in all business schools. A similar but more rapid pattern of adoption is occurring with real options. Ten years ago very few companies used real options, but a recent survey of CFOs reported that more than $26 \%$ of companies now use real option techniques when evaluating projects. ${ }^{9}$ Just as with NPV, it's only a matter of time before virtually all companies use real option techniques.

[^11]FIGURE 25-8 Estimating the Input for Stock Return Variance in the Growth Option Analysis (Millions of Dollars)


## Notes:

${ }^{\text {a }}$ The WACC is $14 \%$. The Year-3 through Year-4 cash flows are discounted back to Year 2.
${ }^{\mathrm{b}}$ The standard deviation is calculated as in Chapter 6.
${ }^{\text {c }}$ The coefficient of variation is the standard deviation divided by the expected value.
${ }^{\mathrm{d}}$ The Year-0 price is the expected PV from Figure 25-7.
${ }^{\text {e }}$ The Year-2 PVs are from Part 1.
${ }^{f}$ The returns for each scenario are calculated as $\left(\mathrm{PV}_{\text {Year }} / \text { Price }_{\text {Year } 0}\right)^{0.5}-1$.
${ }^{9}$ The expected 1 -year return is not equal to the cost of capital, $14 \%$. However, if you do the calculations then you'll see that the expected 2 -year return is $14 \%$ compounded twice, or $(1.14)^{2}-1=29.26 \%$.
${ }^{\text {h }}$ The variance of return is the standard deviation squared.
'The expected "price" at the time the option expires is taken from Part 1.
iThe standard deviation of the expected "price" at the time the option expires is taken from Part 1.

FIGURE 25-9 Estimating the Value of the Growth Option Using a Standard Financial Option (Millions of Dollars)


[^12]See Ch25 Tool Kit.xls on the textbook's Web site for all calculations.

We have provided you with some basic tools necessary for evaluating real options, starting with the ability to identify real options and make qualitative assessments regarding a real option's value. Decision trees are another important tool, since they facilitate an explicit identification of the embedded options, which is critical in the decision-making process. However, keep in mind that the decision tree should not use the original project's cost of capital. Although finance theory has not yet provided a way to estimate the appropriate cost of capital for a decision tree, sensitivity analysis can identify the effect that different costs of capital have on the project's value.

Many real options can be analyzed using a standard model for an existing financial option, such as the Black-Scholes model for calls and puts. There are also other financial models for a variety of options. These include the option to exchange one asset for another, the option to purchase the minimum or the maximum of two or
more assets, the option on an average of several assets, and even an option on an option. ${ }^{10}$ In fact, there are entire textbooks that describe even more options. ${ }^{11}$ Given the large number of standard models for existing financial options, it is often possible to find a financial option that resembles the real option being analyzed.

Sometimes there are some real options that don't resemble any financial options. But the good news is that many of these options can be valued using techniques from financial engineering. This is frequently the case if there is a traded financial asset that matches the risk of the real option. For example, many oil companies use oil futures contracts to price the real options that are embedded in various exploration and leasing strategies. With the explosion in the markets for derivatives, there are now financial contracts that span an incredible variety of risks. This means that an ever-increasing number of real options can be valued using these financial instruments. Most financial engineering techniques are beyond the scope of this book, but Web Extension 25B on the textbook's Web site describes one particularly useful financial engineering technique called risk-neutral valuation. ${ }^{12}$

How widely used is real option analysis?
What techniques can be used to analyze real options?

In this chapter we discussed some topics that go beyond the simple capital budgeting framework, including the following.

- Investing in a new project often brings with it a potential increase in the firm's future opportunities. Opportunities are, in effect, options-the right but not the obligation to take some future action.
- A project may have an option value that is not accounted for in a conventional NPV analysis. Any project that expands the firm's set of opportunities has positive option value.
- Real options are opportunities for management to respond to changes in market conditions and involve "real" rather than "financial" assets.

[^13]- There are five possible procedures for valuing real options: (1) DCF analysis only, and ignore the real option; (2) DCF analysis and a qualitative assessment of the real option's value; (3) decision-tree analysis; (4) analysis with a standard model for an existing financial option; and (5) financial engineering techniques.
- Many investment timing options and growth options can be valued using the Black-Scholes call option pricing model.
- See Web Extension 25A at the textbook's Web site for an illustration of valuing the abandonment option.
- See Web Extension 25B at the textbook's Web site for a discussion of riskneutral valuation.


## Questions

(25-1) Define each of the following terms:
a. Real option; managerial option; strategic option; embedded option
b. Investment timing option; growth option; abandonment option; flexibility option
c. Decision tree
(25-2) What factors should a company consider when it decides whether to invest in a project today or to wait until more information becomes available?
(25-3) In general, do timing options make it more or less likely that a project will be accepted today?
(25-4) If a company has an option to abandon a project, would this tend to make the company more or less likely to accept the project today?

## Solution Appears in Appendix A

(ST-1)
Katie Watkins, an entrepreneur, believes that consolidation is the key to profit in the fragmented recreational equine industry. In particular, she is considering starting a business that will develop and sell franchises to other owner-operators, who will then board and train hunter-jumper horses. The initial cost to develop and implement the franchise concept is $\$ 8$ million. She estimates a $25 \%$ probability of high demand for the concept, in which case she will receive cash flows of $\$ 13$ million at the end of each year for the next 2 years. She estimates a $50 \%$ probability of medium demand, in which case the annual cash flows will be $\$ 7$ million for 2 years, and a $25 \%$ probability of low demand with an annual cash flow of $\$ 1$ million for 2 years. She estimates the appropriate cost of capital is $15 \%$. The risk-free rate is $6 \%$.
a. Find the NPV of each scenario, and then find the expected NPV.
b. Now assume that the expertise gained by taking on the project will lead to an opportunity at the end of Year 2 to undertake a similar venture that will have the same cost as the original project. The new project's cash flows would follow whichever branch resulted for the original project. In other words, there would be an $\$ 8$ million cost at the end of Year 2 and then cash flows of $\$ 13$ million, $\$ 7$ million, or $\$ 1$ million for Years 3 and 4. Use decision-tree analysis to estimate the combined value of the original project and the additional project (but implement the additional project only if it is optimal to do so). Assume that the $\$ 8$ million cost at Year 2 is known with certainty and should be discounted at

## Problems

Intermediate Problems 1-5
(25-1)
Investment Timing Option: Decision-Tree Analysis

Kim Hotels is interested in developing a new hotel in Seoul. The company estimates that the hotel would require an initial investment of \$20 million. Kim expects the hotel will produce positive cash flows of $\$ 3$ million a year at the end of each of the next 20 years. The project's cost of capital is $13 \%$.
a. What is the project's net present value?
b. Kim expects the cash flows to be $\$ 3$ million a year, but it recognizes that the cash flows could actually be much higher or lower, depending on whether the Korean government imposes a large hotel tax. One year from now, Kim will know whether the tax will be imposed. There is a $50 \%$ chance that the tax will be imposed, in which case the yearly cash flows will be only $\$ 2.2$ million. At the same time, there is a $50 \%$ chance that the tax will not be imposed, in which case the yearly cash flows will be $\$ 3.8$ million. Kim is deciding whether to proceed with the hotel today or to wait a year to find out whether the tax will be imposed. If Kim waits a year, the initial investment will remain at $\$ 20$ million. Assume that all cash flows are discounted at $13 \%$. Use decision-tree analysis to determine whether Kim should proceed with the project today or wait a year before deciding.
(25-2) The Karns Oil Company is deciding whether to drill for oil on a tract of land the

Investment Timing Option: Decision-Tree
company owns. The company estimates the project would cost $\$ 8$ million today. Karns estimates that, once drilled, the oil will generate positive net cash flows of $\$ 4$ million a year at the end of each of the next 4 years. Although the company is fairly confident about its cash flow forecast, in 2 years it will have more information about the local geology and about the price of oil. Karns estimates that if it waits 2 years then the project would cost $\$ 9$ million. Moreover, if it waits 2 years, then there is a $90 \%$ chance that the net cash flows would be $\$ 4.2$ million a year for 4 years and a $10 \%$ chance that they would be $\$ 2.2$ million a year for 4 years. Assume all cash flows are discounted at $10 \%$.
a. If the company chooses to drill today, what is the project's net present value?
b. Using decision-tree analysis, does it make sense to wait 2 years before deciding whether to drill?
(25-3)
Investment Timing Option: Decision-Tree Analysis

Real Options: DecisionTree Analysis

Hart Lumber is considering the purchase of a paper company, which would require an initial investment of $\$ 300$ million. Hart estimates that the paper company would provide net cash flows of $\$ 40$ million at the end of each of the next 20 years. The cost of capital for the paper company is $13 \%$.
a. Should Hart purchase the paper company?
b. Hart's best guess is that cash flows will be $\$ 40$ million a year, but it realizes that the cash flows are as likely to be $\$ 30$ million a year as $\$ 50$ million. One year from now, it will find out whether the cash flows will be $\$ 30$ million or $\$ 50$ million. In addition, Hart could sell the paper company at Year 3 for $\$ 280$ million. Given this additional information, does decision-tree analysis indicate that it makes sense to purchase the paper company? Again, assume that all cash flows are discounted at $13 \%$.

Utah Enterprises is considering buying a vacant lot that sells for $\$ 1.2$ million. If the property is purchased, the company's plan is to spend another $\$ 5$ million today $(t=0)$ to build a hotel on the property. The after-tax cash flows from the hotel will depend critically on whether the state imposes a tourism tax in this year's legislative session. If the tax is imposed, the hotel is expected to produce after-tax cash inflows of $\$ 600,000$ at the end of each of the next 15 years, versus $\$ 1,200,00$ if the tax is not imposed. The project has a $12 \%$ cost of capital. Assume at the outset that the company does not have the option to delay the project. Use decision-tree analysis to answer the following questions.
a. What is the project's expected NPV if the tax is imposed?
b. What is the project's expected NPV if the tax is not imposed?
c. Given that there is a $50 \%$ chance that the tax will be imposed, what is the project's expected NPV if the company proceeds with it today?
d. Although the company does not have an option to delay construction, it does have the option to abandon the project 1 year from now if the tax is imposed. If it abandons the project, it would sell the complete property 1 year from now at an expected price of $\$ 6$ million. Once the project is abandoned, the company would no longer receive any cash inflows from it. If all cash flows are discounted at $12 \%$, would the existence of this abandonment option affect the company's decision to proceed with the project today?
e. Assume there is no option to abandon or delay the project but that the company has an option to purchase an adjacent property in 1 year at a price of $\$ 1.5$ million. If the tourism tax is imposed, then the net present value of developing this property (as of $t=1$ ) is only $\$ 300,000$ (so it wouldn't make sense to purchase the property for $\$ 1.5$ million). However, if the tax is not imposed, then the net present value of the future opportunities from developing the property would be $\$ 4$ million (as of $\mathrm{t}=1$ ). Thus, under this scenario it would make sense to purchase the property for $\$ 1.5$ million. Given that cash flows are discounted at $12 \%$ and that there's a 50-50 chance the tax will be imposed, how much would the company pay today for the option to purchase this property 1 year from now for $\$ 1.5$ million?

Fethe's Funny Hats is considering selling trademarked, orange-haired curly wigs for University of Tennessee football games. The purchase cost for a 2-year franchise to sell the wigs is $\$ 20,000$. If demand is good ( $40 \%$ probability), then the net cash flows will be $\$ 25,000$ per year for 2 years. If demand is bad ( $60 \%$ probability), then the net cash flows will be $\$ 5,000$ per year for 2 years. Fethe's cost of capital is $10 \%$.
a. What is the expected NPV of the project?
b. If Fethe makes the investment today, then it will have the option to renew the franchise fee for 2 more years at the end of Year 2 for an additional payment of $\$ 20,000$. In this case, the cash flows that occurred in Years 1 and 2 will be repeated (so if demand was good in Years 1 and 2, it will continue to be good in Years 3 and 4). Write out the decision tree and use decision-tree analysis to calculate the expected NPV of this project, including the option to continue for an additional 2 years. Note: The franchise fee payment at the end of Year 2 is known, so it should be discounted at the risk-free rate, which is $6 \%$.
(25-6)
Investment Timing Option: Option Analysis
(25-7)
Investment Timing Option: Option Analysis
(25-8)
Growth Option: Option Analysis

Rework Problem 25-1 using the Black-Scholes model to estimate the value of the option. Assume that the variance of the project's rate of return is $6.87 \%$ and that the risk-free rate is $8 \%$.

Rework Problem 25-2 using the Black-Scholes model to estimate the value of the option. Assume that the variance of the project's rate of return is $1.11 \%$ and that the risk-free rate is $6 \%$.

Rework Problem 25-5 using the Black-Scholes model to estimate the value of the option. Assume that the variance of the project's rate of return is $20.25 \%$ and that the risk-free rate is $6 \%$.

## Spreadsheet Problem

(25-9)
Build a Model: Real
Options

Start with the partial model in the file Cb25 P09 Build a Model.xls on the textbook's Web site. Bradford Services Inc. (BSI) is considering a project with a cost of $\$ 10$ million and an expected life of 3 years. There is a $30 \%$ probability of good conditions, in which case the project will provide a cash flow of $\$ 9$ million at the end of each of the next 3 years. There is a $40 \%$ probability of medium conditions, in which case the annual cash flows will be $\$ 4$ million, and there is a $30 \%$ probability of bad conditions with a cash flow of $-\$ 1$ million per year. BSI uses a $12 \%$ cost of capital to evaluate projects like this.
a. Find the project's expected present value, NPV, and the coefficient of variation of the present value.
b. Now suppose that BSI can abandon the project at the end of the first year by selling it for $\$ 6$ million. BSI will still receive the Year-1 cash flows, but will receive no cash flows in subsequent years.
c. Now assume that the project cannot be shut down. However, expertise gained by taking it on would lead to an opportunity at the end of Year 3 to undertake a venture that would have the same cost as the original project, and the new project's cash flows would follow whichever branch resulted for the original project. In other words, there would be a second $\$ 10$ million cost at the end of Year 3 followed by cash flows of either $\$ 9$ million, $\$ 4$ million, or $-\$ 1$ million for the subsequent 3 years. Use decision-tree analysis to estimate the value of the project, including the opportunity to implement the new project at Year 3. Assume that the $\$ 10$ million cost at Year 3 is known with certainty and should be discounted at the risk-free rate of $6 \%$.
d. Now suppose the original project (no abandonment option or additional growth option) could be delayed a year. All the cash flows would remain unchanged, but information obtained during that year would tell the company exactly which set
of demand conditions existed. Use decision-tree analysis to estimate the value of the project if it is delayed by 1 year. (Hint: Discount the $\$ 10$ million cost at the risk-free rate of $6 \%$ because the cost is known with certainty.)
e. Go back to part c. Instead of using decision-tree analysis, use the Black-Scholes model to estimate the value of the growth option. The risk-free rate is $6 \%$, and the variance of the project's rate of return is $22 \%$.

Assume you have just been hired as a financial analyst by Tropical Sweets Inc., a mid-sized California company that specializes in creating exotic candies from tropical fruits such as mangoes, papayas, and dates. The firm's CEO, George Yamaguchi, recently returned from an industry corporate executive conference in San Francisco, and one of the sessions he attended addressed real options. Because no one at Tropical Sweets is familiar with the basics of real options, Yamaguchi has asked you to prepare a brief report that the firm's executives can use to gain at least a cursory understanding of the topic.

To begin, you gathered some outside materials on the subject and used these materials to draft a list of pertinent questions that need to be answered. Now that the questions have been drafted, you must develop the answers.
a. What are some types of real options?
b. What are five possible procedures for analyzing a real option?
c. Tropical Sweets is considering a project that will cost $\$ 70$ million and will generate expected cash flows of $\$ 30$ million per year for 3 years. The cost of capital for this type of project is $10 \%$, and the risk-free rate is $6 \%$. After discussions with the marketing department, you learn that there is a $30 \%$ chance of high demand with associated future cash flows of $\$ 45$ million per year. There is also a $40 \%$ chance of average demand with cash flows of $\$ 30$ million per year as well as a $30 \%$ chance of low demand with cash flows of only $\$ 15$ million per year. What is the expected NPV?
d. Now suppose this project has an investment timing option, since it can be delayed for a year. The cost will still be $\$ 70$ million at the end of the year, and the cash flows for the scenarios will still last 3 years. However, Tropical Sweets will know the level of demand and will implement the project only if it adds value to the company. Perform a qualitative assessment of the investment timing option's value.
e. Use decision-tree analysis to calculate the NPV of the project with the investment timing option.
f. Use a financial option pricing model to estimate the value of the investment timing option.
g. Now suppose that the cost of the project is $\$ 75$ million and the project cannot be delayed. However, if Tropical Sweets implements the project then the firm will have a growth option: the opportunity to replicate the original project at the end of its life. What is the total expected NPV of the two projects if both are implemented?
h. Tropical Sweets will replicate the original project only if demand is high. Using decision-tree analysis, estimate the value of the project with the growth option.
i. Use a financial option model to estimate the value of the project with the growth option.
j. What happens to the value of the growth option if the variance of the project's return is $14.2 \%$ ? What if it is $50 \%$ ? How might this explain the high valuations of many startup high-tech companies that have yet to show positive earnings?


[^0]:    Sources: Kate Linebaugh, "Honda's Flexible Plants Provide Edge," The Wall Street Journal, September 23, 2008, p. B1.

[^1]:    ${ }^{1}$ For an excellent general discussion of the problems inherent in discounted cash flow valuation techniques as applied to capital budgeting, see Avinash K. Dixit and Robert S. Pindyck, "The Options Approach to Capital Investment," Harvard Business Review, May/June 1995, pp. 105-115.

[^2]:    Notes:
    ${ }^{\text {a }}$ The WACC is $14 \%$.
    ${ }^{\mathrm{b}}$ The standard deviation is calculated as explained in Chapter 6.
    ${ }^{\text {c }}$ The coefficient of variation is the standard deviation divided by the expected value.
    ${ }^{d}$ The NPV in Part 2 is as of Year 0 . Therefore, each of the project cash flows is discounted back one more year than in Part 1.

[^3]:    ${ }^{2}$ For a more detailed explanation of the rationale behind using the risk-free rate to discount the project cost, see Timothy A. Luehrman, "Investment Opportunities as Real Options: Getting Started on the Numbers," Harvard Business Review, July/August 1998, pp. 51-67. This paper also provides a discussion of real option valuation. Professor Luehrman also wrote a follow-up paper that provides an excellent discussion of the ways real options affect strategy: "Strategy as a Portfolio of Real Options," Harvard Business Review, September/October 1998, pp. 89-99.
    ${ }^{3}$ Murphy might gain information by waiting, which could reduce risk; but if a delay would enable others to enter and perhaps preempt the market, this could increase risk. In our example, we assumed that Murphy has a patent on critical components of the device, precluding the entrance of a competitor that could preempt its position in the market.

[^4]:    Notes:
    ${ }^{\text {a The }}$ operating cash flows in Year 2 through Year 4 are discounted at the WACC of $14 \%$. The cost in Year 1 is discounted at the risk-free rate of $6 \%$.
    ${ }^{\mathrm{b}}$ The standard deviation is calculated as explained in Chapter 6.
    ${ }^{\text {c }}$ The coefficient of variation is the standard deviation divided by the expected value.

[^5]:    Notes:
    ${ }^{\text {a }}$ The WACC is $14 \%$. All cash flows in this scenario are discounted back to Year 0 .
    ${ }^{\text {b }}$ Here we find the PV, not the NPV, because the project's cost is ignored.
    ${ }^{\text {c }}$ The standard deviation is calculated as explained in Chapter 6 .
    ${ }^{\mathrm{d}}$ The coefficient of variation is the standard deviation divided by the expected value.

[^6]:    ${ }^{4}$ In theory, financial option pricing models apply only to assets that are continuously traded in a market. Even though real options usually don't meet this criterion, financial option models often provide a reasonably accurate approximation of the real option's value.

[^7]:    ${ }^{5}$ The company itself is not involved with traded stock options. However, if the option were a warrant issued by the company, then the strike price would affect the company's cash flows and hence its stock price.
    ${ }^{\circ}$ Two points should be made about the percentage return. First, for use in the Black-Scholes model, we need a percentage return calculated as shown, not an IRR return. The IRR is not used in the option pricing approach. Second, the expected return comes to $14 \%$, the same as the WACC. This is because the Year- 0 price and the Year- 1 PVs were all calculated using the $14 \%$ WACC and because we measured return over only 1 year. If we measure the compound return over more than 1 year, then the average return generally will not equal $14 \%$.
    ${ }^{7}$ For a more detailed discussion, see David C. Shimko, Finance in Contimuous Time (Miami, FL: Kolb Publishing, 1992).

[^8]:    Notes:
    ${ }^{\text {a }}$ The current value of the project is taken from Figure 25-3.
    ${ }^{\text {b }}$ The variance of the project's rate of return is taken from Part 3 of Figure 25-4.

[^9]:    ${ }^{8}$ Financial engineering techniques are widely used for the creation and valuation of derivative securities.

[^10]:    Notes:
    ${ }^{\text {a }}$ The operating cash flows are discounted by the WACC of $14 \%$.
    ${ }^{\text {b }}$ The standard deviation is calculated as in Chapter 6.
    ${ }^{\text {c }}$ The coefficient of variation is the standard deviation divided by the expected value.
    ${ }^{d}$ The total cash flows at Year 2 are equal to the operating cash flows for the first-generation product minus the $\$ 30$ million cost to implement the second-generation product, if the firm chooses to do so. For example, the Year-2 cash flow in the high-demand scenario is $\$ 34-\$ 30=\$ 4$ million. Based on Part 1, it makes economic sense to implement the second-generation product only if demand is high or average.
    ${ }^{\text {e }}$ The operating cash flows in Year 1 through Year 2, which do not include the $\$ 30$ million cost of implementing the second-generation project at Year 2 for the high-demand and average-demand scenarios, are discounted at the WACC of $14 \%$. The $\$ 30$ million implementation cost at Year 2 for the high-demand and average-demand scenarios is discounted at the risk-free rate of $6 \%$.

[^11]:    ${ }^{9}$ See John R. Graham and Campbell R. Harvey, "The Theory and Practice of Corporate Finance: Evidence from the Field," Journal of Financial Economics, May 2001, pp. 187-243.

[^12]:    Notes:
    ${ }^{\text {a }}$ The current value of the project is taken from Figure 25-7.
    ${ }^{\text {b }}$ The variance of the project's rate of return is taken from Part 3 of Figure 25-8.

[^13]:    ${ }^{10}$ See W. Margrabe, "The Value of an Option to Exchange One Asset for Another," Fournal of Finance, March 1978, pp. 177-186; R. Stulz, "Options on the Minimum or Maximum of Two Risky Assets: Analysis and Applications," Journal of Financial Economics, Vol. 10, 1982, pp. 161-185; H. Johnson, "Options on the Maximum or Minimum of Several Assets," Fournal of Financial and Quantitative Analysis, September 1987, pp. 277-283; P. Ritchken, L. Sankarasubramanian, and A. M. Vijh, "Averaging Options for Capping Total Costs," Financial Management, Autumn 1990, pp. 35-41; and R. Geske, "The Valuation of Compound Options," Journal of Financial Economics, March 1979, pp. 63-81.
    ${ }^{11}$ See John C. Hull, Options, Futures, and Other Derivatives, 7th ed. (Upper Saddle River, NJ: PrenticeHall, 2009).
    ${ }^{12}$ For more on real options, see Martha Amram, Value Sweep: Mapping Corporate Growth Opportunities (Boston: Harvard Business School Press, 2002); Martha Amram and Nalin Kulatilaka, Real Options: Managing Strategic Investment in an Uncertain World (Boston: Harvard Business School Press, 1999); Michael Brennan and Lenos Trigeorgis, Project Flexibility, Agency, and Competition: New Developments in the Theory and Application of Real Options (New York: Oxford University Press, 2000); Eduardo Schwartz and Lenos Trigeorgis, Real Options and Investment Under Uncertainty (Cambridge, MA: MIT Press, 2001); Han T. J. Smit and Lenos Trigeorgis, Strategic Investment: Real Options and Games (Princeton, NJ: Princeton University Press, 2004); Lenos Trigeorgis, Real Options in Capital Investment: Models, Strategies, and Applications (Westport, CT: Praeger, 1995); and Lenos Trigeorgis, Real Options: Managerial Flexibility and Strategy in Resource Allocation (Cambridge, MA: MIT Press, 1996).

